Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Gold Level (Hardest) G4

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
57	50	43	36	29	21

1. Find the exact solutions to the equations

(a) $\ln x + \ln 3 = \ln 6$,

(2)

(b) $e^x + 3e^{-x} = 4$.

(4)

June 2007

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2. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$.

(2)

January 2008

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3. Given that

$$2 \cos (x + 50)° = \sin (x + 40)°.$$

(a) Show, without using a calculator, that

$$\tan x° = \frac{1}{3} \tan 40°.$$

(4)

(b) Hence solve, for $0 \leq \theta < 360$,

$$2 \cos (2\theta + 50)° = \sin (2\theta + 40)°,$$

giving your answers to 1 decimal place.

(4)

June 2013

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4. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

January 2009

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5. Given that

$$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6},$$

(*a*) find $\frac{dx}{dy}$ in terms of y.

(2)

(*b*) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}.$$

(4)

(*c*) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x. Give your answer in its simplest form.

(4)

June 2013

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6. (*a*) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8\sin^3\theta - 6\sin\theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(*b*) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

January 2009

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7. (*a*) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(*b*) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

(ii) Find the value of θ, for $0 \le \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2 \sin\left(\frac{4\pi t}{25}\right) - 1.5 \cos\left(\frac{4\pi t}{25}\right), \quad 0 \le t < 12,$$

where t hours is the number of hours after midday.

(*c*) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(*d*) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

June 2010

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8. Given that

$$\frac{d}{dx}(\cos x) = -\sin x,$$

(*a*) show that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that $x = \sec 2y$,

(*b*) find $\dfrac{dx}{dy}$ in terms of y.

(2)

(*c*) Hence find $\dfrac{dy}{dx}$ in terms of x.

(4)

January 2011

TOTAL FOR PAPER: 75 MARKS

END

6665 Core Mathematics C3 – G1 Mark Scheme

Question Number	Scheme	Marks
1. (*a*)	$\ln 3x = \ln 6$ or $\ln x = \ln\left(\frac{6}{3}\right)$ or $\ln\left(\frac{3x}{6}\right) = 0$	M1
	$x = 2$ (only this answer)	A1 (cso) (2)
(*b*)	$(e^x)^2 - 4e^x + 3 = 0$ (any 3 term form)	M1
	$(e^x - 3)(e^x - 1) = 0$	
	$e^x = 3$ or $e^x = 1$ Solving quadratic	M1 dep
	$x = \ln 3$, $x = 0$ (or ln 1)	M1 A1 (4)
		(6 marks)
2.	(a)	
	$\frac{dy}{dx} = 2e^{2x}\tan x + e^{2x}\sec^2 x$	M1 A1+A1
	$\frac{dy}{dx} = 0 \Rightarrow 2e^{2x}\tan x + e^{2x}\sec^2 x = 0$	M1
	$2\tan x + 1 + \tan^2 x = 0$	A1
	$(\tan x + 1)^2 = 0$	
	$\tan x = -1$ * cso	A1 (6)
	(b) $\left(\frac{dy}{dx}\right)_0 = 1$	M1
	Equation of tangent at $(0, 0)$ is $y = x$	A1 (2)
		[8]

Question Number	Scheme	Marks
3(a)	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$	M1
	$\sin x(\cos 40 + 2\sin 50) = \cos x(2\cos 50 - \sin 40)$	
	$\div\cos x \Rightarrow \tan x(\cos 40 + 2\sin 50) = 2\cos 50 - \sin 40$	M1
	$\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$, (or numerical answer awrt 0.28)	A1
	States or uses cos50 = sin40 and cos40 =sin50 and so $\tan x^{\circ} = \frac{1}{3}\tan 40^{\circ}$ *	A1 cao **(4)**
(b)	Deduces $\tan 2\theta = \frac{1}{3}\tan 40$	M1
	$2\theta =$ 15.6 so $\theta =$ awrt 7.8(1) One answer	A1
	Also $2\theta =$ 195.6, 375.6 , 555.6 fi $\theta =$ ..	M1 A1
	$\theta =$ awrt 7.8 , 97.8, 187.8, 277.8 All 4 answers	
		(4)
		[8]
4.	$x = \cos(2y + \pi)$	
	$\dfrac{dx}{dy} = -2\sin(2y + \pi)$	M1 A1
	$\dfrac{dy}{dx} = -\dfrac{1}{2\sin(2y+\pi)}$ Follow through their $\dfrac{dx}{dy}$ before or after substitution	A1ft
	At $y = \dfrac{\pi}{4}$, $\dfrac{dy}{dx} = -\dfrac{1}{2\sin\frac{3\pi}{2}} = \dfrac{1}{2}$	B1
	$y - \dfrac{\pi}{4} = \dfrac{1}{2}x$	M1
	$y = \dfrac{1}{2}x + \dfrac{\pi}{4}$	A1 (6)
		[6]

Question Number	Scheme	Marks
5(a)	$\frac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y = \left(6\sec^2 3y \tan 3y\right)$ $\left(\text{oe } \frac{6\sin 3y}{\cos^3 3y}\right)$	M1 A1 **(2)**
(b)	Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6\sec^2 3y \tan 3y}$	M1
	$\tan^2 3y = \sec^2 3y - 1 = x - 1$	B1
	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just x.	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$	A1* **(4)**
(c)	$\frac{d^2y}{dx^2} = \frac{0-[6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	M1 A1
	$\frac{d^2y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} \quad = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	M1 A1 **(4)**
		[10]
6 (a)(i)	$\sin 3\theta = \sin\left(2\theta + \theta\right)$	
	$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$	
	$= 2\sin\theta\cos\theta.\cos\theta + \left(1-2\sin^2\theta\right)\sin\theta$	M1 A1
	$= 2\sin\theta\left(1-\sin^2\theta\right) + \sin\theta - 2\sin^3\theta$	M1
	$= 3\sin\theta - 4\sin^3\theta$ * cso	A1 (4)
(ii)	$8\sin^3\theta - 6\sin\theta + 1 = 0$	
	$-2\sin 3\theta + 1 = 0$	M1 A1
	$\sin 3\theta = \frac{1}{2}$	M1
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$	A1 A1 (5)

Question Number	Scheme	Marks
(b)	$\sin 15° = \sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45°$	M1
	$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$	M1 A1
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ * cso	A1 (4)
		[13]

Question Number	Scheme	Marks
7. (a)	$R = \sqrt{6.25}$ or 2.5	B1
	$\tan\alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = \text{awrt } 0.6435$	M1A1
		(3)
(b) (i)	Max Value $= 2.5$	B1 $\sqrt{}$
(ii)	$\underline{\sin(\theta - 0.6435) = 1}$ or $\underline{\theta - \text{their }\alpha = \frac{\pi}{2}}$; $\Rightarrow \theta = \text{awrt } 2.21$	M1;A1 $\sqrt{}$
		(3)
(c)	$H_{\text{Max}} = 8.5$ (m)	B1 $\sqrt{}$
	$\underline{\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1}$ or $\underline{\frac{4\pi t}{25} = \text{their (b) answer}}$; $\Rightarrow t = \text{awrt } 4.41$	M1;A1
		(3)
(d)	$\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$; $\Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$	M1;M1
	$\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41	A1
	Either $t = \text{awrt } 2.1$ or awrt 6.7	A1
	So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \left\{\pi - 0.411517\ldots \text{ or } 2.730076\ldots^{c}\right\}$	ddM1
	Times $= \{14:06,\ 18:43\}$	A1 (6)
		[15]

Question Number	Scheme		Marks
8. (a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both $\underline{\underline{\text{underlined steps.}}}$	A1 (3)
(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4}, n \in ¢$.		
	$\dfrac{dx}{dy} = 2\sec 2y \tan 2y$	$K\sec 2y \tan 2y$	M1
		$2\sec 2y \tan 2y$	A1 (2)
(c)	$\dfrac{dy}{dx} = \dfrac{1}{2\sec 2y \tan 2y}$	Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{2x\tan 2y}$	Substitutes x for $\sec 2y$.	M1
	$1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$	Attempts to use the identity $1 + \tan^2 A = \sec^2 A$	M1
	So $\tan^2 2y = x^2 - 1$		
	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	A1
			(4)
			[9]

Comments from Examiners' Reports:

1. In part (a) statements like $x + 3 = 6$ and $\ln x = \ln 6 - \ln 3 = \ln 3 \ln 6$ were quite common, and even candidates who reached the stage $\ln x = \ln 2$ did not always produce the correct answer of $x = 2$; $x = e^2$ and $x = 1.99..$, from $x = e^{0.693}$, were not uncommon.

However, it was part (b) where so much poor work was seen; the fact that this required to be set up as a quadratic in e^x was missed by the vast majority of candidates.

Grade A candidates averaged 4 out of 6 marks on this question; the overall average was 2.8. This was the toughest starter question set in recent years.

2. In part (a), the majority of candidates were able to handle the differentiation competently and most were aware that their result had to be equated to zero. The subsequent work in part (a) was less well done with relatively few candidates completing the proof.

In part (b), candidates that had found $\frac{dy}{dx}$ correctly in part (b) were usually able to find the gradient and proceed to a correct equation. However, a significant number had their tangent passing through (0, 1) to give an equation of $y = x + 1$. A minority thought that the result $\tan x = -1$ in part (a) implied that the gradient in part (b) was–1..

Grade A candidates averaged 7 out of 8 marks on this question; the overall average was just under 5.

3. In part (a) most candidates were able to expand the expressions correctly, so achieving the first mark. The first three marks were as much as the majority of candidates could obtain as very few recognised the connection between cos 50° and sin 40°, cos 40° and sin 50°.

Part (b) proved to be much more accessible with most candidates making a really good attempt at it and often achieving all four solutions.

Grade A* students managed 6 or 7 out of 8 on this question and grade A students just under 5; however, just under 30% of students only scored 0 or 1 on this question, with 7.8% scoring full marks.

4. This proved a discriminating question on the January 2009 paper. Those who knew the correct method often introduced the complication of expanding $\cos(2y + \pi)$ using a trigonometric addition formula. Among those who chose a correct method, the most frequently seen error was differentiating $\cos(2y + \pi)$ as $-\sin(2y + \pi)$.

An instructive error was seen when candidates changed the variable y to the variable x while inverting, proceeding from $\frac{dx}{dy} = -2\sin(2y + \pi)$ to $\frac{dy}{dx} = -\frac{1}{2\sin(2x + \pi)}$. This probably reflected a confusion between inverting, in the sense of finding a reciprocal, and the standard method of finding an inverse function, where the variables x and y are interchanged.

Grade A* students average 5 marks, grade A students 4 marks. The mean overall was half marks.

5. Though part (a) would appear to be a straightforward use of the chain rule or even differentiation using the product rule, it proved tricky for the majority of candidates with many not seeming to understand that $\sec^2(3y)$ is an alternative way of writing $(\sec 3y)^2$. The derivative of $\sec x$ is also given in the formula booklet.

Most candidates gained the first mark in part (b) for inverting their $\frac{dx}{dy}$. Those who had part (a) correct were generally successful with this part.

Part (c) was perhaps the most challenging aspect of the question. Only the strongest candidates were able to simplify their result to the required expression as a single fraction with a linear numerator.

The modal score on this question was 6 out of 10, which was also the mean score for grade A students.

6. Part (a)(i) was well done and majority of candidates produced efficient proofs.

Many candidates struggled with part (b). If an appropriate pair of angles were chosen, those who used $\sin 45° = \cos 45° = \frac{\sqrt{2}}{2}$ usually found it easier to complete the question than those who used $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$.

The average score on this question was 7.5 out of 13; grade A* students dropped 1 mark on average, grade A students 4 marks.

7. This was the most demanding question on the June 2010 paper and many candidates were unable to apply their successful work in parts (a) and (b) to the other two parts of the question.

In part (a), almost all candidates were able to obtain the correct value of R, although a few omitted it at this stage and found it later on in the question.

In part (b), many candidates were able to state the maximum value.

Only a minority of candidates recognised the need for a second solution and so lost the final two marks.

The average score on this question was 8 out of 15; grade A* students dropped 1 mark on average, grade A students just under 4 marks.

8. This was a 'show that' question and candidates are expected to demonstrate that the answer is true and not simply write it down.

The successful candidates in part (b) used the result in part (a) to simply write down the answer. Marks were lost by candidates who wrote the solution as sec $2y$ tan $2y$, sec $2x$ tan $2x$ or indeed the LHS as $\frac{\mathrm{d}y}{\mathrm{d}x}$.

In part (c) most candidates recognised the need to invert their answer for (b) reaching $\frac{\mathrm{d}y}{\mathrm{d}x} = 1/\frac{\mathrm{d}x}{\mathrm{d}y}$. Many also replaced sec $2y$ by x often stopping at that point.

The average mark on this question was about half-marks; grade A* students generally scored full marks, grade A students 6.5 out of 9.

Statistics for C3 Practice Paper G4

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	6	n/a	47	2.80		4.02	2.54	1.96	1.55	1.21	0.80
2	8	n/a	61	4.87		7.30	6.15	4.83	3.44	2.45	0.98
3	8	1	45	3.59	6.41	4.72	3.62	2.69	1.99	1.32	0.63
4	6		51	3.07		5.19	4.04	2.96	1.89	1.35	0.31
5	10	6	40	4.04	7.64	6.01	4.29	2.75	1.56	0.79	0.34
6	13		58	7.56		11.92	9.15	6.96	4.86	2.38	1.29
7	15		53	8.00	13.85	11.29	8.26	5.43	3.49	2.08	0.94
8	9		48	4.34	8.36	6.57	4.79	3.36	2.43	1.30	0.62
	75		**51**	**38.27**		**57.02**	**42.84**	**30.94**	**21.21**	**12.88**	**5.91**